TYPOLOGY AND FACTOR ANALYSIS

By Dr. E. Taschdjian

INTRODUCTION

In 1941 the writer, in a publication entitled "Typology and Topology" has attempted to sketch the outlines of a procedure which would allow a mathematical formulation of the essences which characterize natural species or similar manifolds, such as varieties, races or types in general. He was then of the opinion, that the proposed procedure was new and original; in the course of other work which has been undertaken in the meantime it was found, however, that quite similar trains of thought are being pursued by other workers and even though these are not identical in all respects with the opinions of the writer, their value in others is so great for all who are interested in the problems of anthropological, ethnological, sociological, biological or psychological classification, that a short outline of these ideas seems justified.

1. THE PROBLEM

a) Its philosophical aspect

Being may be of two kinds: "Being here" or existence and "Being thus" or essence. Existence or spatial being is extensional, since the object which exists occupies a definite extension of space. Essence, on the other hand, is an intensional persistence in time which makes that an object, even though it does not remain "identical" in all respects, still remains the "same" object throughout a certain period. Every natural object, therefore, obtains its reality from two principles, extensional matter and intensional form, with the result that it shows a Being-within-Becoming, a curious aspect of nature the characterization of which has occupied philosophers from the time of the Eleatics down to Husserl's phenomenology.

Besides natural objects, however, there is given in our experience another class of entities which are free of this dualism: These are ideas which remain self-identical throughout time according to their definition and which are not subject to becoming. Thus e.g. whenever I or anybody else thinks of the number "3", this presence within the mind is identically the same and immutable because it has only actual, but no potential qualities. Once this is clearly realized, the assumption is very plausible that the essences of natural objects belong also to the realm of ideas and this assumption, as is well known, constitutes the great synthesis which Plato performed in order to reconcile the eleatic with the heraclitic viewpoint.

To characterize these essential active ideas in such an exact way that the changing character of things would become predictable had been attempted already by the Pythagoreans in Greece and by the compilators of the Yi-King in China and since then the idea of a "mathesis universalis" has never failed to passionate the best philosophical minds of the world. In fact, the elaboration of differential calculus by Newton and Leibniz is the direct result of such an attempt and to it we owe the whole development of technology in the last century. We realize today, though, that differential and integral calculus is only the first successful step and that we require mathematical methods which would allow us to characterize intensional manifolds as a *whole*. It seems that vector and matrix calculus are destined to fulfill this role.

b) Its semantic aspect

Ideas are represented in space by symbols. But the curious feature of this representative relation is, that it is not a strict, functional one-one relation, but a one-many relation. Thus e.g. the same identical idea may be represented by the symbols 3, III, three, trois, drei, tre etc. The matter of which these symbols are composed is irrelevant for this relation, for whether the symbol is composed of graphit dust, ink pigment, chalk or neon tubes does not influence its effect; neither does its size. The spatial configuration of the symbol also is of no importance for its function, for "3" and "three" have a completely different configuration and yet their semantic function is the same. What these different symbols have in common is only their *meaning* and their function consists in nothing else but in the spatial representation of this meaning. It is the underlying idea which constitutes the common ground for the activity of the different symbols and since the common ground of activity of different entities is called in Aristotelian terminology a "topos", the representative relation which constitutes the semantic function of symbols is a topical relation.

It is important to realize that although a picture stands to the object which it symbolizes in a representative relation and although its semantic function will be the more perfect the more closely the picture resembles the object, yet, a complete agreement is precluded by the topical nature of the relation. Already Plato, in his famous dialogue "Kratylos" emphasizes this point when he says that if a god had created a picture of a man which agreed with him not only exteriorily and in parts, but through and through, this would be no real picture at all, but rather the man himself once over.

The pictures and symbols which we humans use, however, are very far indeed from such a perfect agreement with their object and progress consists exactly in perfecting this agreement, so that a more adequate representation may be obtained. This has been specially stressed by Confucius in his doctrine of the "Rectification of Names". There can be no doubt, however, that even the best terminology is less exact than mathematical symbolization. Both language and mathematics are semantic structures and modern mathematicians agree to define their science as the grammar of all symbolic systems[1].

c) Its biometrical aspect

The first task with which we are confronted when we undertake the study of nature is that of classification; even simple observation of qualities presupposes an unconscious classification, for these qualities are not perceived as existing by themselves, but as inherent in an object, which constitutes a class of which the qualities are members. When we say "This is a dog" we simply mean that it is a set of correlated qualities of which some are essential and others accidental. Since "dog-ness" is an abstract class, an essence, it is really incorrect to say "This is a dog"; what we should say is "This represents a dog". The symbol "dog" refers to the underlying idea which constitutes the definition of the species, not to the spatially existing body; if this were otherwise, we should require not only a different name for each individual dog, but even a different name for each moment of the dog's life, for the body at two moments does not remain identical. In other words, just as we have a one-many relation between an idea and its different symbols so we have also a one-many relation between species and their representatives or, in general between classes and their members.

It is this underlying idea which determines the normality or abnormality of a character. Suppose e. g. that we have observed the inhabitants of an Alpine village afflicted with goitre. Since most of the inhabitants will show this disease we should be obliged, if we based our criterion of normality or abnormality merely on the observed average, to consider all afflicted individuals as normal and the healthy ones as

1) Black, M., **The nature of mathematics.** Kegan Paul, Trench, Trubner & Co. Ltd., London 1933, p. 4.

abnormal. It is only by considering them as members of a different class, viz. of humanity, that the abnormality of the feature becomes evident. Normality or abnormality are therefore not characteristics of a measured value as such, but apply only to the relation between a class and its members[2].

Relations between two variable qualities may be either functional one-one relations or correlational one-many relations. Thus, if we have ten dogs and study the variability of their size and weight, we can make out a list of ten pairs of values, indicating the size and weight of each individual. Such a list shows that there is a factual connection between the two members of a pair of values but this does not entitle us to assume that the relation is a functional one, for even though in this list there corresponds to one value of size only one value of weight, there are other dogs of the same size which have different weights. Yet, these corresponding weights are not determined by pure chance, for not every possible weight can correspond to a given size, as long as the entity or class in which the two values occur is still a dog. There must be something, then, in this entity "dog" which restricts the number of possible values of a quality and yet allows them a certain range of variation. This something we conceive as a norm, i. e. a prescriptive law of the type "you should" and thus establish a connection between typology and the philosophy of law. Such connections between two ranges of values are defined mathematically with the help of correlation calculus and through it we are enabled to predict probabilities for the occurrence of a given value within a given set or class.

d) Its ethnological and sociological aspect

Like the biologist and anthropologist, the ethnologist and sociologist also is confronted daily with problems of classification. A comparison of two social groups always yields a number of points in which they agree and a number of others in which they disagree; these latter are then said to characterize the group. But this characterization of a group by a criterium is not of the presence-absence type. Sun gods and sun myths e. g. occur in such widely different ethnical units as Egyptians, Carthaginians, Persians, Mayas, Incas, Japanese etc. Hence the occurrence or non-occurrence of a single criterium is insufficient to characterize an ethnical or social unit and only a number of criteria allow a sufficient distinction. One and the same criterium in different groups or within one group at different times may be of different intensities and may hence occur in different frequencies. Although agriculture may be the rule in

2) Taschdjian, E., **The bionomics of procreation.** The Catholic University Press, Peking, 1942, p. 108.

a peasant type of civilization, this does not mean that occasionally a hare or a duck will not be shot there; even though smokers usually ride in smoking-cars and non smokers in other railway cars, this does not mean that occasionally a non-smoker may not be found in a smoking car or vice versa[3]. Just as in biometry, therefore, ethnological and sociological typology allows us predictions only with regard to probabilities of events and hence here also correlation calculus must be applied if we wish to evaluate the probabilities of social events accurately.

e) *Its psychological aspect*

Psychology is interested primarily in the characterization of human individuals in order to make predictions for their behaviour. For this purpose, a great number of tests which purport to measure intelligence, will power, sensitivity, attention etc. have been devised. None of these tests, taken singly, however is able to give us a sufficient reliability, for it is senseless to say that the intelligences of Wallenstein: Rossini: Newton: Rubens: Goethe are as 11: 12: 13: 14: 19. The more tests we apply, the better the characterization will become, but we must not forget that "scientia est de universalibus" and that therefore psychology which attempts to enumerate *all* the traits of an individual, like e. g. the differential psychology of W. Stern, is an anamnesis, but no science. Just as a picture which agrees in all points with its model is no picture any more, but the object once over, so also a psychological classification which can predict human behaviour with absolute certainty is an impossibility; we must attempt greater exactitude, but only within topological limits and psychological predictions also must be predictions of probabilities.

The different qualities which we measure by psychological tests, however, are not independent of each other. If an individual ranks very high in one intelligence test it is improbable that he will rank very low in another one and the same applies when we compare tests of intelligence with tests of attention or coordination or memory. Hence we find here also the phenomenon that even though more than one value of one quality corresponds to a given value of another, the possible number of such correspondences is not infinite but is restricted to a certain range by an underlying law or, to express it still more generally, by something which the two variables have in common. The analysis of these "common factors" is the purpose of mathematical typology and of factor analysis of which we shall attempt now a short description.

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3) Menger, K., Einige neuere Fortschritte in der exakten Behandlung sozialwissenschaftlicher Probleme, in: **Neuere Fortschritte in den exakten Wissenschaften.** F. Deuticke, Leipzig & Wien, 1936, pp. 125-126.

2. THE SOLUTION

a) First approach

The writer conceives the "common factors" mentioned above as an intensional manifoldness of a prescriptive type. If I call to a man "come here", the latter may or may not obey, but compared to a man to whom no such order is given, the probabilities that he will move in the direction ordered, are increased. Consequently prescriptive laws are factors which influence the probabilities of events even though they are not of a physical or extensional nature. The basic problem of typology reduces itself thus to the comparison of different laws of distribution for variables and such a comparison, if it is to be exact, i. e. if it is to be a measurement, requires that the law be exhaustively characterized by a description a) of its intensity and b) of its form.

Ad a) The intensity of a law of distribution is best measured by the obedience which it finds in the relata for which it constitutes the common ground of activity. This obedience expresses itself in the probabilities of the events and consequently a suitable measure of intensity must be based exclusively on the estimation of probabilities. Such an intensional measure of correlations is available in Pearson's "Mean square contingency", represented by the formula

$$\varphi^2 = \sum_{i=1}^{k} \sum_{j=1}^{l} \frac{[P_{i1j} - P_{i1} P_{1j}]^2}{P_{i1} P_{1j}}$$

In this formula p_{i1} represents the probability that the variable X assumes the value X_i out of its k possible values and p_{1j} represents the probability that the variable Y will assume out of its l possible values the value Y_j; p_{i1j} means the probability that simultaneously X will assume the value X_i and Y the value Y_j. The value φ^2 of this measure of intensity of a correlation varies between -1.0 and $+1.0$, just as with the more commonly used Bravais coefficient of correlation. Here as there a value of -1.0 means a perfect negative correlation and a value of $+1.0$ means a perfect positive correlation. In these two cases there corresponds to one value of X one and only one value of Y, i. e. we have then a strict functional connection. These are two border cases which in all those fields which interest us at present practically never occur. Usually we shall find values intermediate between 1.0 and 0, which mean that more than one value of X corresponds to a given value of Y. If, on the other hand, the intensity of correlation becomes zero, this means that the two variables are independent of each other and that any value of X can correspond to a given value of Y.

Ad b) A form is a whole which is the common ground of the activities of its parts *as* parts. A class of students e. g. is such a whole, the parts of which are students, not men or women. What characterizes students as students is that they conform to certain prescriptions which allow them certain alternatives of action and forbid others. Prescriptions are represented in mathematics by a class of symbols which are called "operators". Thus, in the prescription "add a to b", the form of the prescription is represented by the symbol + in the formula a + b. Other operators are such signs as $\sqrt{\ }$, Σ, $|\ \ |$, etc. These operators represent the relation as a whole, of which the relata are the parts.

Suppose we study in dynamics two forces with different directions acting on a body. The forces are represented graphically by two vectors, the length of which corresponds to their intensities whilst their direction is shown by arrowhead signs. (Fig. 1) The point where the two vectors intersect or in other words the body which they both attack is the one thing which they have in common, it is their common "topos". Now, mathematically, the curve of movement produced by a vector acting on a body is expressed by an equation. If two vectors act on a body, we have two simultaneous equations to solve and that part of their activity which they have in common is expressed mathematically by the *determinant* of the two equations. Determinants again are parts of a higher class of mathematical operators, viz. of *matrices*, which are therefore the wholes of which determinants are the parts.

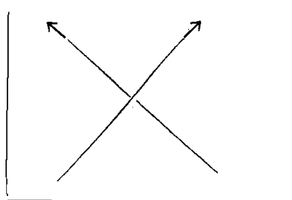

Fig. 1.

A static (*) type of whatsoever nature is a whole of a certain number of qualities which are correlated in certain intensities; these intensities we have characterized above by their mean square contingencies. Each correlation, however, does not exist by itself, but only as part of the type as a whole which constitutes for them their common topos. Consequently we can describe the type as a whole mathematically by a matrix which embodies the totality of all coefficients of contingency, as follows:

*) The problem of the transformations of types in time, which is of great importance for the biologist and the historian, but also for educational psychology, falls outside the scope of the present article. See on this point (4). Taschdjian, E., **The bionomics of procreation**, loc. cit., 119-124.

$$\left\| \begin{matrix} \varphi_{ab}^{|p|}, & \varphi_{ac}^{|p|}, & \varphi_{ad}^{|p|} & \cdots\cdots & \varphi_{an}^{|p|} \\ \varphi_{ba}^{|p|}, & \varphi_{bc}^{|p|}, & \varphi_{bd}^{|p|} & \cdots\cdots & \varphi_{bn}^{|p|} \\ \vdots & & & & \\ \varphi_{na}^{|p|}, & \varphi_{nb}^{|p|}, & \varphi_{nc}^{|p|} & \cdots\cdots & \varphi_{n(n-1)}^{|p|} \end{matrix} \right\|$$

In this matrix a....n are the n qualities which are correlated within the type. The index $|p|$ represents the actual intensity of the respective correlation, i. e. it is a number varying from 0 to + 1.0.

b) Second approach

P. R. Hofstätter, basing himself mainly on previous works of Thurstone gives an outline of a method developed by the latter which is called "Factor Analysis". This method of analysis differs from that outlined above mainly in that it neglects and ignores the topological and intensional character of the correlations and types analyzed. But on the other hand the mathematical apparatus which was only sketched by the writer, is fully developed by the Chicago school of psychologists and its value for typology can hardly be overemphasized. The description given by Hofstätter of this method, on which the following outline will be based, differs from the original procedure of Thurstone in that it avoids the use of matrix calculus in order to simplify the matter for laymen. For the same reason we shall follow him here, rather than the original, although in the subsequent discussion both publications will be considered.

Let us suppose now that a number N of individuals are subjected to a number j of psychometric tests s. The efficiency of a given individual i in the test j be dependent upon the possession of two abilities 1 and 2. The performance of this individual will then be represented by the equation

$$s_{ji} = x_{1i} + x_{2i} \qquad (1)$$

provided that the two abilities contribute equally to the efficiency. Since this need not be the case we have to introduce further weight factors a_{j1} and a_{j2} indicating the contribution of each ability to the total performance. Equation (1) is thus transformed into

$$s_{ji} = a_{j1}\,x_{1i} + a_{j2}\,x_{2i} \qquad (2)$$

and if a number q of factors or abilities contribute to the efficiency, we get

$$s_{ji} = a_{j1}\,x_{1i} + a_{j2}\,x_{2i} + \cdots\cdots a_{jq}\,x_{qi} \qquad (3)$$

An analogous equation applies to the efficiency of the same individual in the test k:

$$s_{ki} = a_{k1}\, x_{1i} + a_{k2}\, x_{2i} + \cdots\cdots a_{kq}\, x_{qi} \qquad (3a)$$

The two tests j and k are not independent of each other, but have an internal connection or correlation which is given by the equation

$$r_{jk} = (a_{j1}\ a_{k1} + a_{j2}\ a_{k2}) \qquad (4)$$

and for q common factors we have again

$$r_{jk} = (a_{j1}\ a_{k1} + a_{j2}\ a_{k2} + \cdots\cdots a_{jq}\ a_{kq}) \qquad (4a)$$

In both (4) and (4a)

$$r_{jk} = \frac{1}{N} \sum_{i=1}^{n} s_{ji}\ s_{ki} \qquad (5)$$

is the Bravais-Pearson coefficient of correlation. Expressed in words equation (4a) means that the correlation of the two tests j and k is equal to the sum of the products of the fractions with which the two tests participate in the common factors.

Since this coefficient of correlation varies between -1.0 and $+1.0$ it is natural to represent it graphically in the form of an angle, resp. of a geometric function. It is the cosinus which is especially adapted to this purpose, since this varies also between -1.0 and $+1.0$. We assume therefore for each test a unit vector, the correlation of which is represented by the cosinus of the angle which they form with each other:

$$r_{jk} = \cos(jk)$$

and this is represented geometrically by Fig. 2

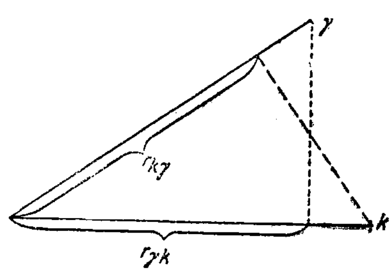

Fig. 2. Geometric interpretation of correlations. (From Hofstätter)

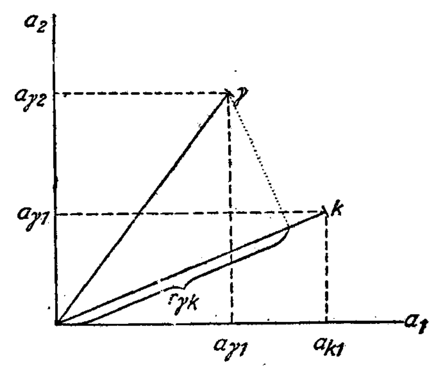

Fig. 3. Representation of a correlation in a two-dimensional system. (From Hofstätter)

The weight factors a_{j1}, a_{j2} can also be interpreted geometrically. They may be regarded either as the cosines of the angle formed by the respective test vector with the a_1 -resp. a_2 -axis or as the coordinates of the terminal point of the vector in its projection on the two axes. (See Fig. 3) The weight factors a_{j1}, a_{j2} are hence nothing but the correlations of test j with the contributing factors a_1 resp. a_2 and these are interpreted as coordinates:

$$r_{ja1} = a_{j1};\ r_{ja2} = a_{j2}$$

The definition and calculation of these axes $a_1, a_2 \ldots . a_q$ is the first problem of factor analysis, but this can be solved only if the values $a_{j1} \ldots . a_{jq}$ for the single tests have already been determined. This is done in the following way:

If we add the totality of correlations which a test k has with all other tests, we obtain

$$\sum_{j=1}^{n} r_{jk} = a_{k1} \sum_{j=1}^{m} a_{j1} + a_{k2} \sum_{j=1}^{m} a_{j2} + \cdots\cdots = r_k \qquad (6)$$

If we carry out such summations for the total number m of tests and add the values thus obtained, the final sum of all correlations r_t is given by

$$\sum_{j=1}^{m} \sum_{k=1}^{m} r_{jk} = \sum_{k=1}^{m} a_{k1} \sum_{j=1}^{m} a_{j1} + \sum_{k=1}^{m} a_{k2} \sum_{j=1}^{m} a_{j2} + \cdots\cdots = r_t \qquad (7)$$

Since each correlation table corresponds to a diagonally symmetric matrix ($r_{jk} = r_{kj}$), we obtain

$$\sum_{k=1}^{m} a_{kq} = \sum_{j=1}^{m} a_{jq} \qquad (8)$$

and equation (7) thus passes into the simpler form

$$\sum_{j=1}^{m} \sum_{k=1}^{m} r_{jk} = (\sum_{j=1}^{m} a_{j1})^2 + (\sum_{j=1}^{m} a_{j2})^2 + \cdots\cdots = r_t \qquad (9)$$

It is from the values r_k and r_t that the weight factors $a_{j1} \ldots . a_{jq}$ can be determined by the so-called centroid method of Thurstone in the following way:

Since tests j and k have been defined as unit vectors and are represented geometrically as diagonals of rectangles (see Fig. 3), there applies

$$a_{j1}^2 + a_{j2}^2 = 1 \tag{10}$$

and for q dimensions, we obtain similarly

$$a_{j1}^2 + a_{j2}^2 + \cdots\cdots a_{jq}^2 = 1 \tag{10a}$$

A series of 4 tests (j, k, s, t) can thus be represented in a two-dimensional coordinate system as shown by Fig. 4.

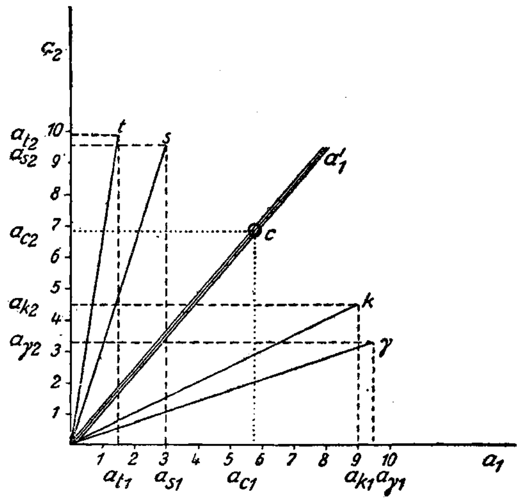

Fig. 4. Determination of the Centroid. (From Hofstätter)

By forming the average of the projections from the end points of the test vectors upon the two coordinates we obtain the abscissa and the ordinate of a point C which is called the center of gravity or centroid of the system.

The coordinates of this point are given by

$$a_{c1} = \frac{1}{m} \sum_{j=1}^{m} a_{j1} \tag{11}$$

and by

$$a_{c2} = \frac{1}{m} \sum_{j=1}^{m} a_{j2} \tag{11a}$$

In a q-dimensional space the center of gravity is defined similarly by the coordinates

$$C \equiv \left(\frac{1}{m} \sum_{j=1}^{m} a_{j1};\ \frac{1}{m} \sum_{j=1}^{m} a_{j2};\ \cdots\cdots\ \frac{1}{m} \sum_{j=1}^{m} a_{jq}\right) \tag{12}$$

The center of gravity is a point around which a system may be turned arbitrarily without affecting its internal structure. In order to isolate the weight factors $a_{j1} \ldots a_{jq}$ therefore, all that we have to do is to turn our coordinate system in such a way that one of its axes will pass through the point C. The new axes of the coordinate system be then a'_1 and a'_2 and for the centroid all coordinates except one thus become equal to zero:

$$C \equiv \frac{1}{m} \sum_{j=1}^{m} a'_{j1} \tag{13}$$

$$\sum_{j=1}^{m} a'_{j2} = \sum_{j=1}^{m} a'_{jq} = O \tag{14}$$

Consequently equations (6) and (7) pass now into the simpler forms

$$\sum_{j=1}^{m} r_{jk} = a'_{k1} \sum_{j=1}^{m} a'_{j1} = r_k \tag{15}$$

and

$$\sum_{j=1}^{m} \sum_{k=1}^{m} r_{jk} = (\sum_{j=1}^{m} a'_{j1})^2 = r_t \tag{16}$$

The two values r_t and r_k can be obtained by simple addition from every correlation table and from them we obtain directly a'_{k1} :

$$r_k = a'_{k1} \sqrt{r_t} \tag{17}$$

$$a'_{k1} = \frac{r_k}{\sqrt{r_t}} \tag{18}$$

In this way the axial segments of tests $k = 1 \ldots m$ can be calculated. The further procedure repeats the precedent steps on the basis of the following arguments. If the correlation of the two tests j and k is caused by a single common factor, we should have according to equation (14)

$$r_{jk} = a'_{j1} \,.\, a'_{k1} \tag{19}$$

and therefore

$$r_{jk} - a'_{j1} \,.\, a'_{k1} = 0 \tag{20}$$

If this equation is not fulfilled, it would indicate that equation (19) is insufficient for an explanation of the correlation r_{jk}, in other words it would mean that more than one common factor is involved. We therefore form for all tests the values (r_{jk} -a'_{j1} . a'_{k1}) and thus obtain a second correlation table, to which we apply, if the values are not all equal to zero, the previous procedure by forming again the r_k -and r_t -values and calculating according to (17) and (18) the values a'_{k2}. This procedure is kept up until it is found that the original correlations are represented with sufficient exactitude by the calculated factor products.

For the practical work Thurstone has formulated a number of prescriptions, regarding which we must refer the reader to the original publications[5, 6].

3. DISCUSSION

It is really surprising that two people, working in different sciences and basing themselves on different philosophical viewpoints should nevertheless come to substantially the same solutions. If anything, this should disprove the nominalistic thesis that classes and categories are purely man-made inventions and have no degree of correspondence with natural reality[7]. Let us list shortly these points of agreement and also those of disagreement.

The two fundamental concepts on which the whole procedure is based are the concepts of Action and Potentiality which, as is well-known, are the two basic categories of Aristotelian philosophy. Thurstone's procedure aims at a mathematical formulation of "primary mental abilities" and he conceives an ability as a trait which is defined by what an individual *can do**. The definition of potential actions is therefore the main object of study[8]. Similarly, the writer conceived the underlying specific essence of a type as a norm prescribing the *actualization* of n qualities[9] and hence the matrix which represents the type represents it only with regard to its abilities or potentialities.

The scope of the enquiry is not limited to any specific form of types or to any specific science. The introductory description of the different aspects of the problem will be sufficient to show that typological enquiries

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5) Thurstone, L. L., **The vectors of mind.** The University of Chicago Press, 1935, pp. 232-250.

6) Hofstätter, P., **Über Faktoren-Analyse.** Archiv für die gesamte Psychologie, Bd. 100, 1938, pp. 276-279.

7) Thurstone, L. L., **The vectors of mind,** loc. cit., p. 44.

8) Ibid., p. 48.

9) Taschdjian, E., **Typology and Topology.** The Catholic University Press, Peking 1941, p. 19.

*) Italics by the writer.

are of a general philosophical nature. Although the writer applied typology mainly to biological and Thurstone mainly to psychological problems, the latter has also expressly stated that factor analysis is applicable also to problems which involve the attributes of inanimate members of a statistical group[10].

The fact that factor analysis aims at a definition of *common* factors and that topology is the science of factors which are common to the actions of natural entities should make it evident that factor analysis is a topological enquiry. Thurstone does not seem to have realized this point. But in Hofstätter's account mention is made of a publication by K. Lewin, called "Principles of topological psychology"[11] and this seems to indicate that this author at least has realized the topological nature of vectors and matrices. Hofstätter rejects a topological interpretation on the assumption that Lewin's vectors are not mathematical concepts and that his topological spaces and dimensions are not of a metric nature, but the writer suspects that some nominalistic preconceptions are responsible for this rejection. Lewin's publication unfortunately was not accessible to the writer and therefore it is impossible to say how far the agreement goes in this respect.

The primary common factors are called in Thurstone's terminology also "communalities"[12]. These are said to vary between 0 and + 1[13], in other words, the assumption is made that all abilities can be expressed in the form of a positive vector. Thus e. g. a negative grouchiness can be expressed as a positive cheerfulness. Similarly, in order to avoid negative potentialities, the writer, although in the first publication on this subject this point was not considered, has introduced later instead of the algebraic value of the correlation intensity the actual intensity in the form of the exponent $|p|$ of the mean square contingencies, varying between 0 and + 1[14].

As can be seen from the matrix of contingency factors written in the "first approach", this matrix is n-dimensional, i. e. it contains n (n-1) correlation coefficients; since $\varphi ab = \varphi ba$ the number of different coefficients in the matrix is n (n-1)/2. Now Thurstone has emphasized[15] that even though this solution is satisfactory as long as the factor problem is regarded only in its purely mathematical aspects, it is unsatisfactory in its application to a concrete problem, because it assumes as many degrees of freedom

10) Thurstone, L. L., **The vectors of mind**, loc. cit., p. 48.
11) Lewin, K., **Principles of topological psychology**, New York, 1936.
12) Thurstone, L. L., **The vectors of mind**, loc. cit., p. 85.
13) Ibid., pp. 73, 165-166.
14) Taschdjian, E., **The bionomics of procreation**, loc. cit., p. 118.
15) Thurstone, L. L., **The vectors of mind**, loc. cit., pp. 77-78.

in the hypothesis as there are experimental observations; this violates the postulate of scientific economy, that a valid hypothesis is overdetermined by the data. This point had not occurred to the writer and its legitimacy must be conceded.

The stroke of genius of Thurstone was the introduction of the centroid vector as the common topos of the single unit vectors. This allows us to rotate the reference axes without any effect on the inter-correlations and permits a practical solution of the matrix without a prohibitive amount of work. The idea of a center of gravity as a characteristic for a vector structure had not occurred to the writer. It is an idea which is more liable to occur in connection with material structures than with intensional ones. For the latter, the writer had proposed a solution by the definition of a tensor[16]. In fact, he has a feeling -which he cannot at present prove mathematically -that Thurstone's centroid vector is really no vector but a tensor. But this, even if it should prove to be true, is really only a minor terminological point and does not detract from the immense practical value of the centroid method.

Another point in which the writer emphatically agrees with Thurstone is in the rejection of the idea of a proportionality between the underlying common factors and their effects[17]. This point has been the subject of controversy in psychology for the last 30 years, because the Spearman school conceived the matrix as of rank 1, which involves the idea of proportionality[18]. The typological matrix proposed by the writer has been expressly stated to be of rank 2[19]. The idea of proportionality is intimately connected with strict, mechanistic causality and involves the idea of necessity in the sense of Laplace. For this reason it is absolutely unfitted for any holistic approach to organic problems.

For the same reason the writer must disagree with Thurstone and Hofstätter with regard to their attempt at interpreting the unitary common factors as Mendelian genes[20] or as genetic linkages[21]. It has been shown in a previous publication, that the materialistic interpretation given by the Morgan school to genetic linkage phenomena is far from adequate and that there occur linkages also between genes located in different chromosomes[22]. Consequently the genic set must be considered

16) Taschdjian, E., **Typology and Topology**, loc. cit., p. 20.

17) Taschdjian, E., **The bionomics of procreation**, loc. cit., pp. 27, 29, 30, 88, 113, 159, 216.

18) Thurstone, L. L., **The vectors of mind**, loc. cit., pp. 134-135.

19) Taschdjian, E., **Typology and Topology**, loc. cit., p. 21.

20) Thurstone, L. L., **The vectors of mind**, loc. cit., p. 206.

21) Hofstätter, P., **Uber Faktoren-Analyse**, loc. cit., pp. 230-232.

22) Taschjian, E., **The bionomics of procreation**, loc. cit., pp. 201, 204-205.

as a whole and communalities are not resultants of the activity of single genes or even single chromosomes, but of the whole configuration of factors. This theoretical postulate does not preclude the possibility, that for practical purposes it may be useful to distinguish those factors which carry a greater weight from those, the contribution of which may be neglected in a first approach. In other words, we are free to consider them as a "Gestalt" of a lower hierarchical order than the genic set as a whole and because of this lower hierarchical order its intensity will be comparatively greater, just as a genus is a "Gestalt" of a lesser intensity than a concrete species.

SUMMARY

1) A survey is made of the philosophical, semantic, biometrical, ethnological, sociological and psychological aspects of the typological problem.
2) An outline is given of a mathematical solution of this problem based on topological considerations.
3) The method of factor analysis is shortly sketched.
4) The topological and factorial methods are compared with regard to their points of agreement and disagreement.